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PERTURBED SPECTRUM TECHNIQUE FOR WAVE PROPAGATION IN PLANAR OPTICAL WAVEGUIDES

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ملخص – طريقة التغير الطفيف للتوافقيات فى هذا البحث مشتقة من طريقة التغيرالطفيف لأنتشار الموجات فى الدلائل الضوئية ، والطريقة المستخدمة تحول المعادلة القياسية للموجات الى مجموعة مترابطة من المعادلات التفاضلية كدالة فى المجال المستعرض والتى يمكن حلها لأيجاد المجال الكهربى والمغناطيسى وذلك بأستخدام نظرية فورير ، والطريقة الموضحة فى هذا البحث يمكن تطبيقها لاى توزيع للعازل المصنوع من دلائل الموجه وأيضا لأى نوع من التغذية لهذه الدلائل ،
ولقد تم أختبار مدى دقة الطريقة بمقارنتها بالطرق الاخرى لدراسة المرابط الموجه ودليل الموجه ذو العاذل المتغير على مراحل أو ذو العاذل المتغير المستمر ، وكانت النتائج فى هذه الاحوال مطابقة بالنتائج المستخلصة من الطرق الاخرى سواء العددية أو التحليلية ،

ABSTRACT- The Perturbed Spectrum Technique (PST) presented in this paper is a perturbation-based method for wave propagation in optical waveguides. The method reduces the scalar wave equation to a couple of ordinary differential equations, which can be solved using the spatial Fourier transform of the transverse distribution of the propagating field. The method can deal with arbitrary refractive index profile as well as arbitrary excitation. The validity of the PST is checked through the study of directional coupler, graded index waveguide and step discontinuity in a planar guide. The results obtained by that method fairly agree with those obtained by other numerical and analytical methods.

INTRODUCTION

We believe that the numerical solution of the wave equation is a crucial factor in deciding wether the relevant theoretical treatment is valuable or not. One of the most widely used methods: the beam-propagation method (BPM) [1-9] requires small variations in the refractive index distribution. Recently, Kumar et. al [4]. introduced a method based on a double Fourier transform applied twice to the propagating field: transverse to the direction of propagation and in the direction of propagation. Thus the problem is reduced to a system of algebraic equations for the unknown double Fourier transform which can be solved by matrix manipulations. A major problem in their method occurs if the propagating field has some periodicity in the direction of propagation, because it will be reflected as sharp giant spikes in the Fourier transform taken along the direction of propagation. These spikes are very offensive in numerical calculations; and that is why Kumar et. al. used a fictitious exponentially decaying function to reduce the amount of these spikes. The rate of decay of that function plays a crucial role in the accuracy of the computations. Unfortunately Kumar et. al. did not give a general criterion for an adequate choice of the rate of decay of that fictitious function. However, their results [4] are satisfactory.

In this paper we present a perturbation-based formulation of the propagation problem that leads finally to a couple of ordinary differential equations for the transverse spatial Fourier spectrum of the unperturbed and the perturbed part of the spectrum of the total propagating field. The solutions of these two equations are calculated numerically using the discrete Fourier transform which can be calculated by the well known Fast Fourier Transform (FFT) algorithm.

1- THEORY

Most of the integrated optical components are fabricated from optical waveguides which have small refractive index varaiations, and hence the paraxial approximation is very adequate to describe the wave propagation in such waveguides. For simplicity, let us consider a planar optical waveguide in which the propagation is in the z - direction, the

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propagating field is confined in the x - direction only, and the waveguide is y - invariant. Let a monochromatic electric field e_y be polarized in the y - direction, and has a time-dependence exp $(-i\ \omega\ t)$, where ω is the angular frequency. Assume that the refractive index profile n(x) of the waveguide can be written as:

$$n\ (x) = n_o + \alpha\ n_1(x) \tag{1}$$

where α is a small dimensionless parameter, which can be taken for example as the maximum variation of n (x) relative to the constant refractive index n_o.

Let us write e_y as:

$$e_y\ (x,z,t) = E\ (x,z).\ \exp\ (ikz - iwt) \tag{2}$$

where k is the wave number in a homogeneous medium whose refractive index is n_o. Modifications in e_y due to the propagation in the inhomogeneous medium n(x) are contained in E (x,z). The two dimensional wave equation for e_y is :

$$\frac{\partial^2 e_y}{\partial x^2} + \frac{\partial^2 e_y}{\partial z^2} + k_o^2\ \ n^2(x)\ e_y = 0 \tag{3}$$

When (2) is substituted in (3), and the second partial z-derivative of E is neglected we obtain:

$$\frac{\partial^2 E}{\partial x^2} + i2k\ \frac{\partial E}{\partial z} - k^2 E + k_o^2\ n^2(x)\ \ E=0 \tag{4}$$

where k_o is the free-space wave number, and $k = k_o\ n_o$.

Seeking a solution of E as the sum of two fields: the first one E_o corresponds to a field propagating in a homogeneous medium (i.e. when $n(x) = n_o$) and the other field e_1 considered as a perturbation to E_o, thus we write:

$$E\ (x,z) = E_o\ (x,z) + e_1\ (x,z) \tag{5}$$

where E_o satsfies (4) when $n\ (x) = n_o$, that is:

$$\frac{\partial^2 E_o}{\partial x^2} + i2k\ \frac{\partial E_o}{\partial z} = 0 \tag{6}$$

Expanding e_1 as a power series in the small parameter α:

$$e_1\ (x,z) = \alpha\ E_1(x,z) + \alpha^2\ E_2(x,z) + \ldots\text{etc.} \tag{7}$$

Substituting the first order term, i.e. $\alpha\ E_1$, in (5), then substituting $E = E_o + \alpha\ E_1$ in (3), and neglecting terms of order α^2, we obtain the following inhomogeneous wave equation for the first-order perturbation E_1:

$$\frac{\partial^2 E_1}{\partial x^2} + i2k\ \frac{\partial E_1}{\partial z} = -\ \frac{2k^2}{n_o}\ n_1(x)\ E_o(x,z) \tag{8}$$

When (6) is transversly Fourier-transformed (i.e. with respect to x), we get:

$$-k_x^2\ \psi_o\ (k_x\ ,\ z) + i2k\ \frac{d\psi_o}{dz}\ = 0 \tag{9}$$

where:

$$\psi_o\ (k_x,z) = \int_{-\infty}^{\infty} E_o(x,z)\ \exp\ (-ik_x x)dx \tag{10}$$

is the transverse spatial Fourier transform of E_o and k_x is the transverse spatial frequency (the variable of the transform). Equation (9) is a first-order ordinary differential equation with parameter k_x, it has the solution:

$$\psi_o\ (k_x,z) = \psi_o(k_x,0)\ \exp\ (-ik_x^2\ z/2k) \tag{11}$$

where $\psi_o\ (k_x,0)$ is the transverse spatial Fourier spectrum of E_o at $z = 0$. Similarly, when (8) is Fourier-transformed we get:

$$-k_x^2\ \psi_1\ (k_x,z) + i2k\ \frac{d\psi_1}{dz}\ = -F(k_x,z) \tag{12}$$

where:

$$\psi_1\ (k_x,z) = \int_{-\infty}^{\infty} E_1(x,z)\ \exp\ (-ik_x x)dx \tag{13}$$

is the perturbation in the Fourier spectrum of the total propagating field, while the source term $F(k_x,z)$ in (12) is:

$$F(k_x, z) = \frac{2k^2}{n_o} \int_{-\infty}^{\infty} n_1(x)\, E_o(x,z) \exp(-ik_x x)\, dx$$

$$= \frac{2k^2}{n_o} N_1(k_x) \otimes \psi_o(k_x, z) \qquad (14)$$

where $N_1(k_x)$ is the Fourier spectrum of $n_1(x)$, and $\otimes$ denotes the convolution of N_1 and ψ_o with respect to k_x. If the total field $E(x,z)$ starts propagation at $z = 0$, then the perturbation part E_1 vanishes at $z = 0$, and as far as the propagation takes place, the field E_o will be accordingly modified by E_1. Thus; the solution of (12) with the initial condition $\psi_1 = 0$ at $z = 0$ taking into account (11) is [5]:

$$\psi_1(k_x,z) = ik_o e^{-ik_x^2 z/2k} \int_0^z \left\{ N_1(k_x) \otimes \psi_o(k_x,\xi) \right\} e^{+ik_x^2 \xi/2k}\, d\xi \qquad (15)$$

We can write (15) in a more compact form by denoting $\phi(k_x,\xi) = N_1 \otimes \psi_o$ in the integrand, so we can write:

$$\psi_1(k_x, z) = ik_o e^{-ik_x^2 z/2k} \int_0^z \phi(k_x, \xi)\, e^{+ik_x^2 \xi/2k}\, d\xi \qquad (16)$$

This equation, together with (11) constitute the solution of the problem.

2- NUMERICAL SOLUTION

The use of the discrete Fourier transform (DFT) enables us to calculate the ψ_o in (11) and ϕ in (16), and hence the integral can be calculated using any of the well known methods: trapezoidal, Simpson's rule or Romberg method. The integration in (16) is to be calculated over a small propagation step Δz (i.e. from 0 to Δz). ψ_1 is inverse-Fourier transformed to obtain the perturbation part $E_1(x,\Delta z)$ of the total propagating field. Then, $\psi_1(k_x,\Delta z)$ is calculated from (11) since the spectrum of the initial field at $z = 0$ is known, and hence the propagation process of $E_o(x,0)$ is reduced to a simple multiplication of $\psi_o(k_x,0)$ by the propagator $\exp(-ik_x^2 \Delta z/2k)$. The field $E_o(x,\Delta z)$ is easily obtained by inverse-Fourier transforming $\psi_o(k_x,\Delta z)$. Finally, $E_1(x,\Delta z)$ is multiplied by the small parameter α and the result added to $E_o(x,\Delta z)$ to yield the total field $E(x,\Delta z)$. That field will be considered as the initial unperturbed field E_o for the next propagation step and so on.

Concerning the evaluation of the integral in (16), we expected that Romberg method will allow to take relatively large Δz, but we did not remark any significant improvement in the step size, and almost identical results are obtained using trapezoidal, Simpson's or Romberg method. This is because the accuracy of the evaluation of the integral in (16) depends strongly on how fast the exponent in the integral in (16) varies. As a creterion, we require that the argument of the exponential do not vary by no more than π over one step at the spatial frequency corresponding to the highest significant component in the source term $\phi(k_x,\xi)$, and hence the proper choice of Δz depends on the extent of the Fourier spectrum of the source term ϕ in the K_x- domain. Accordingly Δz must satisfy the following inequality:

$$\Delta z \le (\lambda\, n_o/4)\, \hat{k}_x^2 \qquad (17)$$

Where $\hat{k}_x$ is the spatial frequency corresponding to the highest significant component in the spectrum of the source term ϕ.

An advantage of the PST presented in this paper, is that we do not need the solution of a system of algebraic equations which may requires matrix inversion, and hence no problems concerning stability or singularities are faced. Furthermore, we do not need any fictitious function like that one needed in Kumar et.el.[4] approach, however we need to put an absorber at the boundary of the computational window to prevent the reflection of the high-frequency component of the spectrum of the propagating field from the boundary of the computational window. This is a well known technique which is often used in any Fourier transform-based approach like the BPM[1-9]. It is worthy to note that while the BPM requires small index-varitions, no such requirement is essential in the perturbed spectrum technique (PST) proposed in this paper.

3- RESULTS

To test our method, we considered some previously studied problems: the propagation in graded index waveguide, and a pair of coupled waveguides, as well as the radiation loss in a step discontinuity in planar waveguides.

a- Graded Index Guide

The focusing of a uniform illumination in a truncated parabolic-index waveguide was demonstrated by Kumar et. al.[4] using a double transform technique when the refractive index profile takes the following distribution:

$$n^2(x) = \begin{cases} n_g^2\,(1-x^2\,\Delta), & |x| < d/2 \\ n_s^2\,, & |x| \geq d/2 \end{cases} \quad (18)$$

where d is the core thickness and,

$$\Delta = 4(n_g^2 - n_s^2)\,/n_g^2\,d^2 \quad (19)$$

Kumar et.al. [4] studied this problem when d = 20 μm, n_g = 2.155, n_s = 2.1398 and the wave-length λ = 1.32 μm. They found that focusing occurs after a propagation distance nearly equal to 140 μm. Figure 1, shows the field evolution each 15.5 μm, where the focusing effect is easily remarked.

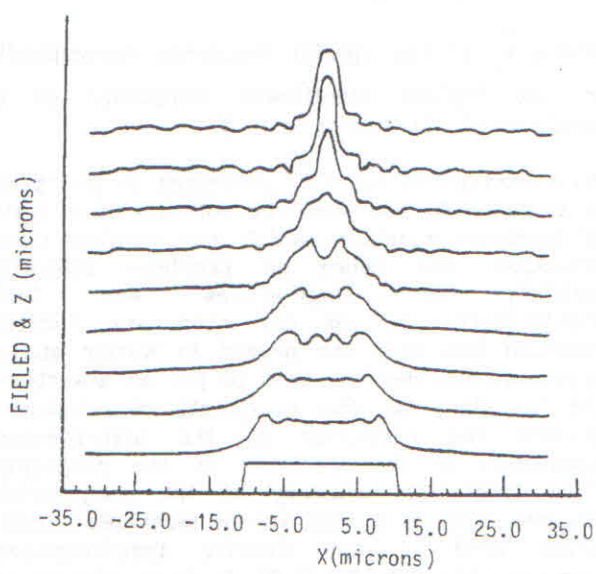

Fig. 1 Focusing of uniform illumination propagating in a parabolic index planar optical waveguide.

b- Coupled Waveguides

The field evolution in a pair of coupled planar waveguides separated by 0.4 μm was investigated by Kumar et.el. [4], when each guide has a thickness 4 μm and a refractive index 2.155 inside the guides and 2.1398 everywhere outside the guides, the wavelength λ = 1.32 μm. The coupled-mode theory predicts a coupling length equal to 290 μm. Figure 2 shows the field evolution each 32 μm when the right guide is excited with the fundamental mode. It can be seen that almost complete power transfer from the left guide to the right guide occurs after 290 μm.

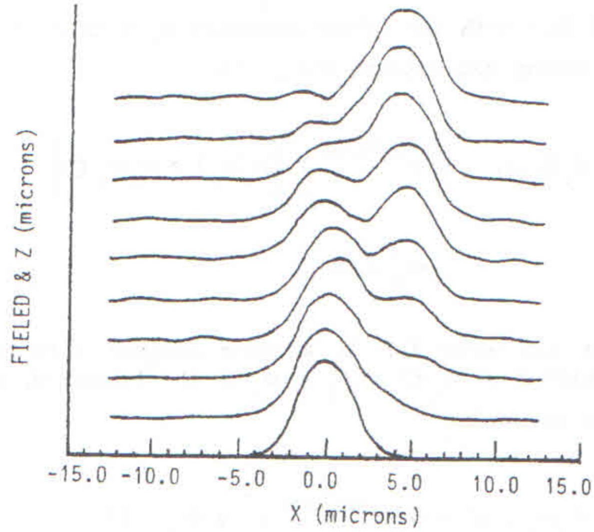

Fig. 2 Power transfer between a pair of coupled waveguides.

c- Step Discontinuity

The power loss by forward radiation at the junction between two single-mode symmetrical planar waveguides (as shown in the inset of figure 3) was studied previously by many authors [6-9].The left waveguide is excited with its fundamental mode. The ratio d_1/d_2 is kept constant equal to 0.5 while $k_o\,d_1$ is varied. The radiated power P_r relative to the power incident from the left of the step reaches a minimum value [6-9] at $k_o\,d_1 \simeq$ 23, λ = 1.57, n_g = 1.01 and n_s = 1.0 (data from references 7-9). We calculate the total propagating field E(x,z) far enough to the right of the junction plane using our method: the PST. To extract the modal field from the total propagating field E(x,z), we can

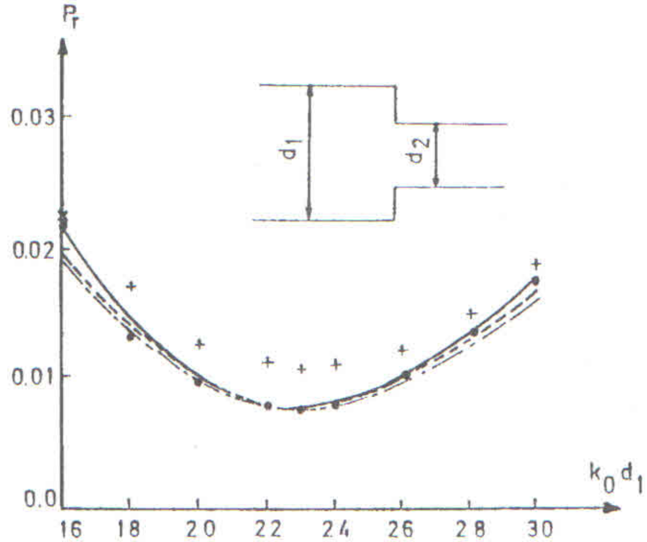

Fig. 3 Relative radiation loss P_r at a step discontinuity
———— Integral equation method
—·—·— Residue - calculus method
-------- Mode - matching method
• Present method
+ Beam propagation method

expand [10] the latter one in terms of the eigenmodes of the planar waveguide located at the right of the step thus we can write:

$$E(x,z) = t_o \varphi_o (x) e^{-i\beta z} + R \tag{20}$$

where φ_o (x) is the transverse distribution of the fundamental mode of the single-mode waveguide at the right of the step, β is the propagation constant, R is the Fourier integral representing the total radiation field and t_o is the excitation coefficient of the fundamental mode. The scalar product of both sides of (20) with the complex conjugate φ_o^* exp ($i\beta z$) gives:

$$t_o = \frac{\int_{-\infty}^{\infty} E(x,z).\ \varphi_o^* dx}{\int_{-\infty}^{\infty} |\varphi_o|^2 dx} \tag{21}$$

The guided transmitted power is readily obtained once t_o is calculated and hence the relative radiated power P_r is easily obtained since it is the difference between the incident power and the guided power transmitted across the junction. To test the accuracy of our method, P_r is calculated as a function of $k_o d_1$ in the vicinity of the point of minimum radiation loss. We compared our results (solid dots on figure 3) with those obtained by very accurate methods: the integral equation method [7] (continuous curve in figure 3), residue-calculus method [8] (dotted curve in figure 3) and the mode-matching method [9] (dashed curve in figure 3), while the crosses present the results obtained by the BPM [6]. It can be seen that our method is quite accurate since it gives a minimum radiation loss very close to the accurate value 0.007, while the BPM gives a minimum radiation loss of 0.011.

4- CONCLUSION

In this paper we propose a method that can be applied to graded index planar guides of arbitrary profile as well as arbitrary excitation. Generalization to three dimensional waveguides is possible if the transverse Fourier transform is substituted with a Hankel transform to take into account the two dimensional confinement of the propagating field.The results obtained by our method fairly agree with those obtained with other numerical and analytical methods. We believe that the method can also be applied to waveguides exhibiting nonlinearities if these nonlinearities can be considered as perturbations, and in this case, the source term of the nonhomogeneous wave equation govering the propagation of the perturbed part of the total propagating field, (r. h. s. of (8)), will be proportional to $|E_o|^2$, or any other power of the nonperturbed part E_o. Even if the r.h.s. of (8) includes terms proportional to the perturbed part E_1 of the total field, then the solution will be an Volterra-type integral equation which can be solved by many well known techniques.

The numerical accuracy of the method relative to other methods is checked through the calculation of the radiation loss at a step discontinuity in symmetrical planar waveguides.

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